function, therefore, f(x) is bijection.

9. $\sum_{k=10}^{100} k = 10 + 11 + \cdots + 100 + 55$

$$= \frac{(100+10)(100-10)}{2} + 55 = 4950 + 55 = 5005$$

(reason, $10 + 100 = 11 + 99 = 12 + 98 = \cdots\cdots = 110$)

and there are total 45 pairs 110 and additional a 55)

8. (a).

$f(x) = x^5$

$f'(x) = x^4$.

because $f'(x) \geq 0$, to $\forall x \in R$

Therefore, $f(x)$ is monotonic function.

So it's bijection

(b). $f(x) = \dfrac{x}{x^2+1}$

$$f'(x) = \frac{x'(x^2+1) - 2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}.$$

$\forall x \in [-1, 1]$ $f'(x) > 0$.

$\forall x < -1$ or $x > 1$. $f'(x) < 0$.

in different interal, $f(x)$ is monotonic function, therefore, $f(x)$ is bijection.

7(d).

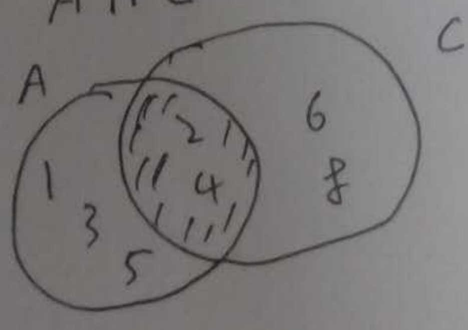
suppose $A = \{1.2.3.4.5\}$.

$B = \{2.3.4.5.6\}$.

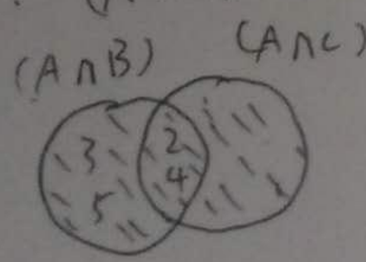
$C = \{2.4.6.8\}$.

step 1: $A \cap B$

step 2: $A \cap C$



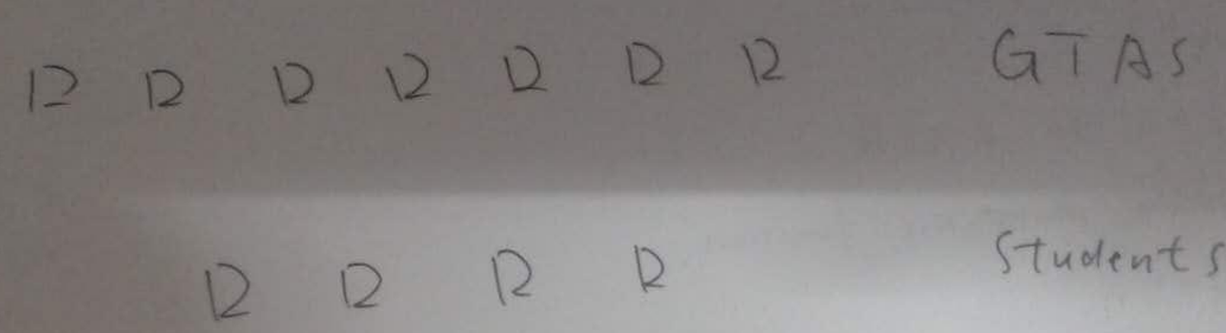
step 3: $(A \cap B) \cup (A \cap C)$



$35\times4\times3\times2\binom{7}{3}$

140×6

11. (a).



According to the requirement. we can

convert to select 4 GTAS in 7 GTAS,

and 4 students work with a GTA.

Then, we got : $\binom{7}{4}4! = 840$.

(b). The ~~probability~~ ways that all the students work

Then, we got : $\binom{7}{4}4! = 840$.

(b). The ~~probability~~ ways that all the students work don't on left are: $\binom{6}{4}4! = 360$

The possibility is : $\dfrac{\binom{6}{4}4!}{\binom{7}{4}4!} = \dfrac{3}{7}$

(c). if a student choose GTA S : the probability is: $\frac{1}{7}$. four students choose GTA S: the probability is: $(\frac{1}{7})^4$.

And the GTA S can be 1, 2, 3 ... 7,

Therefore, the probability is : $(\frac{1}{7})^4 \cdot 7 = \frac{1}{343}$

12. There are sixteen possible outcomes in the Sample Space.

There are 4 possible outcomes for $H \times H \times$

$\times$ can be H or T.

There are 4 possible outcomes for $T \times T \times$.

$\times$ can be H or T

Therefore, the probability $P = \frac{4+4}{16} = \frac{1}{2}$

13. (a). We can list a Table

	1	2	3	4	5	6
					6	7

x can be 4 or 1

Therefore, the probability $p = \frac{4+4}{16} = \frac{1}{2}$

13. (a). We can list a table

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

From the table we can see there are 10 outcomes that the sum is at least 9.

Then: $P(A) = \frac{10}{36} = \frac{5}{18}$

(b). From the table we can get there are 9 outcomes that both blue die and red die are odd.

Then $P(B) = \frac{9}{36} = \frac{1}{4}$

(c). $A \cap B$ only have 1 outcome $(5,5)$,

Therefore $P(A \cap B) = \frac{1}{36}$

(d). because $P(A \cap B) \neq 0$. therefore. A and B are not independent.

(e). $P(A|B)$ means, at the condition that the event B happen, what's the probability of A happen, The outcome of B have 9 outcomes. And there is only 1 outcome that satisfy A.

Then we got $P(A|B) = \frac{1}{9}$

15. Euclid's algorithm as follows:

step 1: $130 = 78 \times 1 + 52$

$r = 52$

step 2: $78 = 52 \times 1 + 26$

$r = 26$

step 3: $52 = 26 \times 2 + 0$

$r = 0$

$\Rightarrow$ the greatest common divisor of 78 and 130 is 26.

probabili~~ty are~~ : $\frac{1}{7^4} \times 7$

11. (d) . There are total 7^4 ways for Students work in GTAS.

The probability that ~~A~~ GTA work with 4 students is: $\frac{1}{7^4} \times 7 = \frac{1}{7^3}$

The probability that a GTA work with 3 students is: $\frac{1}{7^3} \times \frac{6}{7} \times 7 = \frac{6}{7^3}$

The probability that a GTA work with 2 students is: $\frac{1}{7^2} \times \frac{6}{7} \times \frac{5}{7} \times 7 = \frac{30}{7^3}$

The probability that a GTA work with 1 students is: $\frac{1}{7} \times \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times 7 = \frac{1}{2}$

The sum probability is: $\frac{1}{7^1} + \frac{6}{7^3} + \frac{30}{7^3} + \frac{1}{2} = \frac{417}{686}$

1.

p	q	result
1	0	0
0	1	1
0	0	0
1	1	0

2. We know for the proposition:

$p \to q$. it's equal to $\neg p \vee q$.

Therefore. $((p \wedge q) \to r) \vee (r \to q)$

can convert to: $(\neg(p \wedge q) \vee r) \vee (\neg r \vee q)$

~~Whether r=0 or 1, we can conclude~~

if $r=1$, we can see $(\neg(p \wedge q) \vee r)$ is true.

and the total proposition is true.

if $r=0$, we can see $(\neg r \vee q)$ is true.

and the total proposition is true.

Therefore, proposition $((p \wedge q) \to r) \vee (r \to q)$

is a tautology

~~Whether r=0 or 1. we can~~

if $r=1$. we can see $(\neg(p\wedge q)\vee r)$ is true.

and the total proposition is true.

if $r=0$. we can see $(\neg r\vee q)$ is true.

and the total proposition is true.

Therefore, proposition $((p\wedge q)\rightarrow r)\vee(r\rightarrow q)$

is a tautology

2. (b). $p\leftrightarrow q$ is logically equivalent to

$$(p\rightarrow q)\wedge(q\rightarrow p)$$

$(p\rightarrow q)\wedge(q\rightarrow p)$ can convert to:

$$(\neg p\vee q)\wedge(\neg q\vee p)$$

For the above proposition:

it's true if and only if $p=0$. $q=0$ or $p=1$, $q=1$

which is equivalent to $p\leftrightarrow q$.

3. (a). the statement is true:

we can find a value $n = \frac{1}{2}$.

$(\frac{1}{2})^2 < \frac{1}{2}$.

(b). the statement is false:

we can find a value $n = 0$,

$0^2 = 0$ but not $0^2 > 0$.

4. (a). To the statement $\exists m \exists n (n+m=2 \wedge n-m=4)$.

For the domain is the positive real number.

It's impossible that $m, n \in R^{+}$. and satisfy with

$\begin{cases} n+m=2 \\ n-m=2 \end{cases}$ at the same time, therefore, the statement

is false.

(b). The statement is true, because $m, n \in R^{+}$.

Then. $m>0$, $n>0$, then $m+n \neq 0$.

5. It's a valid argument.

There are two premises.

$\forall x(P(x) \vee (Q(x)).\ \exists x(\neg P(x))$.

From the ~~statement~~ premise $\exists x(\neg P(x))$.

we can find a x, that $P(x)$ is false.

So to arbitury x, for the first premise.

$Q(x)$ should always be true.

and the conclusion is: $\exists x \neg (Q(x) \rightarrow P(x))$

we can convert to: $\exists x(Q(x) \wedge \neg P(x))$

$Q(x)$ is alway true, and the premise

$\exists x(\neg P(x))$ is true. we can conclude that:

$\exists x \neg (Q(x) \rightarrow P(x))$ is a valid argument.

$\exists x \neg (Q(x) \to P(x))$ is a valid argument.

~~8~~ 9

6 (a). to $\forall n \in \mathbb{Z}$, the final decimal of n

could be $0, 1, 2 \ldots, 9$.

and to n^2. we calculate $0^2, 1^2, 2^2 \ldots 9^2$,

and get to $\forall n$, the final decimal digit

of n^2 can only be $0, 1, 4, 5, 6, 9$.

Therefore, for n^4, we calculate $0^2, 1^2, 4^2, 5^2, 6^2, 9^2$.

we get the final decimal of n^4 is in the set $\{0, 1, 5, 6\}$,

(b). we can get a example to disprove that the

product of two irrational numbers can be rational number

For example: π, and $\frac{1}{\pi}$ are both irrational number,

but their product is rational number 1.

6. (c).

to $n=1$, we proved $g_1 = 1 = 2^1 = 1$.

suppose to $n=k$. we have $g_k = 2^k - 1$.

To $n=k+1$. ~~we~~ from the formula

$g_n = 2g_{n-1} + 1$, we got $g_{k+1} = 2g_k + 1 = 2(2^k - 1) + 1$

$= 2^{k+1} - 1$, so we can induct that

$g_n = 2^n - 1$

(d). when $n = 8$. $8 = 3\times 1 + 5\times 1$

(d). when $n=8$, $8=3\times1+5\times1$

$n=9$ $\quad 9=3\times3+5\times0$

$n=10$ $\quad 10=3\times0+5\times2$.

$n=11$ $\quad 11=3\times2+5\times1$

$n=12$ $\quad 12=3\times4+5\times0$

$n=13$ $\quad 13=3\times1+5\times2$

$n=14$ $\quad 14=3\times3+5\times1$

$n=15$ $\quad 15=3\times5+5\times0$

$n=16$ $\quad 16=3\times2+5\times2$

to $n\geq17$. we can express them by the value from $n=8 \ldots\ldots 16$.

Therefore, the statement is true.

... 16.

Therefore, the statement is true.

7 (a)

7 (b). the elements are: $-6, -4, -2, 0, 2, 4, 6$

(c). suppose $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5\}$,

$C = \{2, 4, 6, 8\}$.

to left part: $(B \cup C) = \{2, 3, 4, 5, 6, 8\}$.

$A \cap (B \cup C) = \{2, 3, 4\}$.

to right part: $(A \cap B) = \{2, 3, 4\}$.

$(A \cap C) = \{2\}$.

$(A \cap B) \cup (A \cap C) = \{2, 3, 4\}$. $\Rightarrow A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

10.

(a). $A = [1, 2]$, $B = [2, 3]$ are two uncountable set, and they are finite.

(b). $A = N^+ = \{1, 2, 3, 4 \ldots\}$,

$B = Z = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3 \ldots\}$.

(c). $A = R$, A is real number set

$B = R^+$ B is positive real number set